Period 1, April 12, 2024

$F(x) = 8x^3$

$g(x) = \sin \pi x$

(a)

Tangent to F at $x = \frac{1}{2}$

$F(\frac{1}{2}) = 8 \ (\frac{1}{2})^3 = 8 \cdot \frac{1}{8} = 1$

$F(\frac{1}{2}) = 1$ Point $(\frac{1}{2}, 1)$

$F'(x) = 8 \cdot 3 x^{3-1} = 24x^2$

$F'(\frac{1}{2}) = 24 (\frac{1}{2})^2 = 24 \cdot \frac{1}{4} = \frac{24}{4} = 6$ ✓

$m = 6$

$y = mx + b$

$y = 6x + b$

$1 = 6(\frac{1}{2}) + b$

$1 = 3 + b$

$-2 = b$

$y = 6x - 2$ ✓

$y - y_1 = m(x - x_1)$

$y - 1 = 6(x - \frac{1}{2})$ $+1$

$+1$

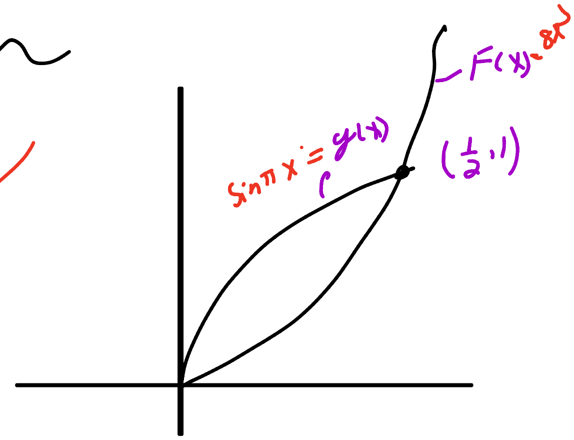
$y = 6(x - \frac{1}{2}) + 1$

Same point

b.

$F(x) = 8x^3$

$g(x) = \sin \pi x$



$\int_0^{\frac{1}{2}} [\sin \pi x - 8x^3]\, dx$ ✓

$\int_0^{\frac{1}{2}} \sin \pi x\, dx - \int_0^{\frac{1}{2}} 8x^3\, dx$

$\int \sin u\, du \quad - \quad 8 \int_0^{\frac{1}{2}} x^3\, dx$

$-\cos u\, du$

$u = \pi x \qquad -8 \cdot \frac{1}{4} \cdot x^{3+1=4}$

$du = \pi dx$

$\frac{du}{\pi} = dx$

$\int \sin u \cdot \frac{du}{\pi}$

$\frac{1}{\pi} \int \sin u\, du$

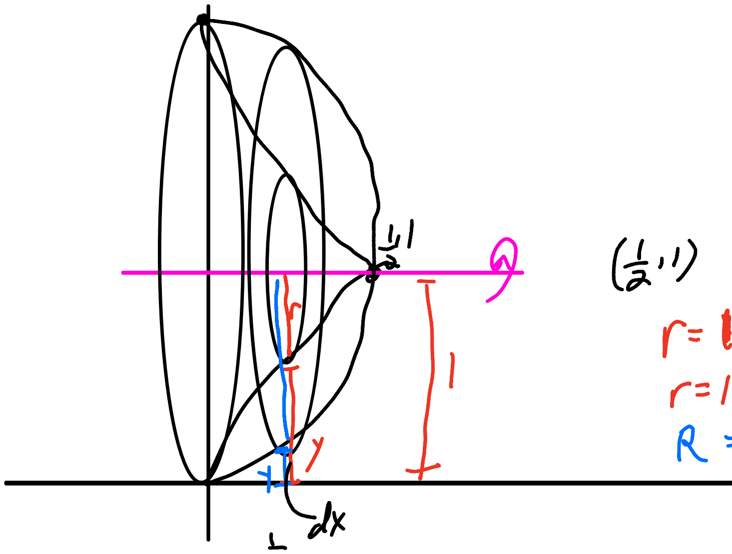
$\frac{1}{\pi} \cdot -\cos u$

$-\frac{1}{\pi} \cos \pi x - 2x^4 + C \Big[_0^{\frac{1}{2}}$ ✓

$-\frac{1}{\pi} \cos \pi \cdot \frac{1}{2} - 2\left(\frac{1}{2}\right)^4 - \left[-\frac{1}{\pi} \cos \pi \cdot 0 - 2(0)^4\right]$

$-\frac{1}{\pi} \cos \frac{\pi}{2} - 2 \cdot \frac{1}{16} + \frac{1}{\pi} \cos 0 + 0$

$-\frac{1}{\pi} \cdot 0 - \frac{1}{8} + \frac{1}{\pi} \cdot 1 + 0 = \frac{1}{\pi} - \frac{1}{8}$ ✓



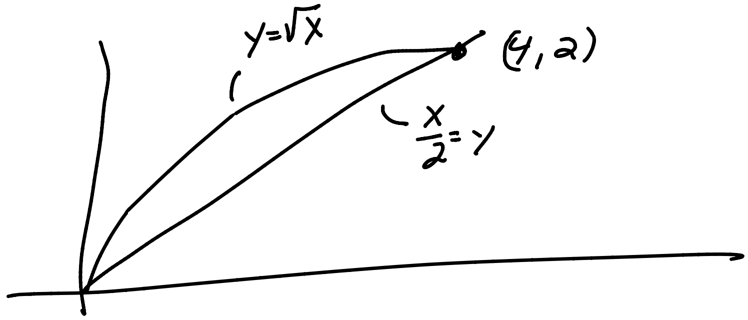
$(\frac{1}{2},1)$

$r = 1 - y$

$r = 1 - \sin \pi x$

$R = 1 - 8x^3$

$$\int_0^{\frac{1}{2}} \pi \left((1-8x^3)^2 - (1-\sin \pi x)^2 \right) dx$$ 3PTS ✓



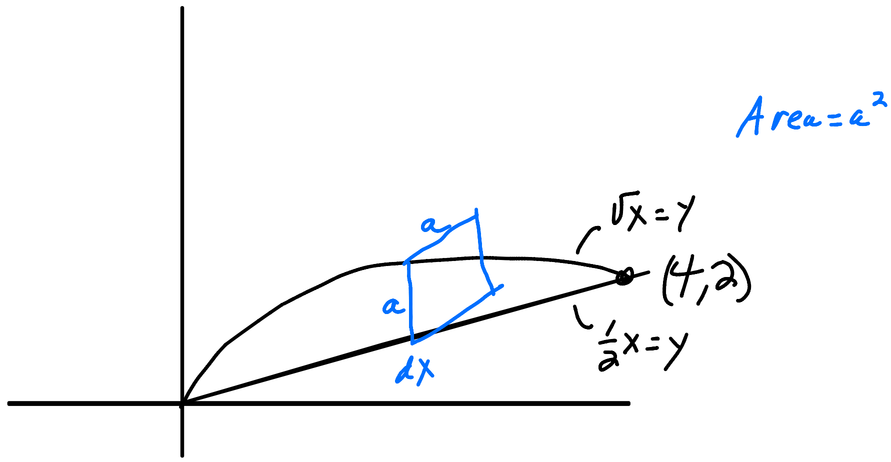
$$\int_0^4 (\sqrt{x} - \frac{x}{2})\, dx$$ ✓

$$\int_0^4 (x^{\frac{1}{2}} - \frac{1}{2} \cdot x)\, dx$$

$$\frac{2}{3} x^{\frac{1}{2}+1 = \frac{3}{2}} - \frac{1}{2} \cdot \frac{1}{2} \cdot x^{1+1=2} + c \Big|_0^4$$ ✓

$$\frac{2}{3} \cdot \sqrt{x^3} - \frac{1}{4}(x)^2 = \frac{2}{3}\sqrt{4^3} - \frac{1}{4}(4)^2 - \left[\cancel{\frac{2}{3}(0)^{\frac{3}{2}} - \frac{1}{4}(0)^2} \right]$$

$$\frac{2}{3} \cdot 8 - 4 = \frac{16}{3} - \frac{12}{3} = \frac{4}{3}$$ ✓



$$\int_0^4 \left(\sqrt{x} - \tfrac{1}{2}x\right)^2 dx$$

$$\left(\sqrt{x} - \tfrac{1}{2}x\right)^2$$

$$\left(\sqrt{x} - \tfrac{1}{2}x\right)\left(\sqrt{x} - \tfrac{1}{2}x\right)$$

$$x - \tfrac{1}{2}x \cdot \sqrt{x} - \tfrac{1}{2}x\sqrt{x} + \tfrac{1}{4}x^2$$

$$\int_0^4 \left(x - \frac{x\sqrt{x}}{2} - \frac{x\sqrt{x}}{2} + \frac{1}{4}x^2\right) dx$$

$$\int_0^4 \left(x - \frac{\cancel{2}x\sqrt{x}}{\cancel{2}} + \frac{1}{4}x^2\right) dx$$

$$\int_0^4 \left(x - x^{\frac{3}{2}} + \frac{1}{4}x^2\right) dx$$

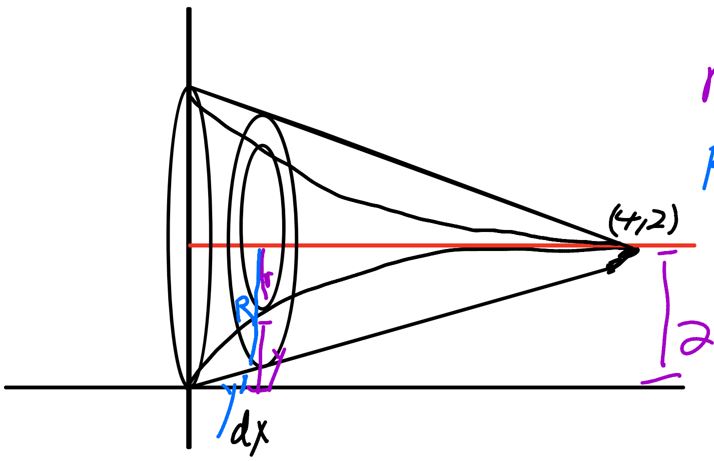
$$\frac{1}{2}x^{1+1=2} - \frac{2}{5}x^{\frac{3}{2}+1=\frac{5}{2}} + \frac{1}{4}\cdot\frac{1}{3}\cdot x^{2+1=3} + c \Big[_0^4$$

$$\frac{1}{2}x^2 - \frac{2}{5}x^{5/2} + \frac{1}{12}x^3 \Big[_0^4 = \frac{1}{2}(4)^2 - \frac{2}{5}(4)^{5/2} + \frac{1}{12}(4)^3 - \left[\frac{1}{2}(0)^2 - \frac{2}{5}(0)^{5/2} + \frac{1}{12}(0)^3\right]$$

$$\frac{16}{2} - \frac{2}{5}(32) + \frac{1}{12}\cdot 64 = 8 - \frac{64}{5} + \frac{64}{12}$$

$$8-\frac{64\cdot 3}{5\cdot 3}+\frac{16\cdot 5}{3\cdot 5}=8-\frac{192}{15}+\frac{80}{15}$$

$$8-\frac{112}{15}=\frac{120-112}{15}=\frac{8}{15}$$



$r=2-y=2-\sqrt{x}$

$R=2-y=2-\frac{1}{2}x$

$$\int_0^4 \pi\left[\left(2-\frac{x}{2}\right)^2-\left(2-\sqrt{x}\right)^2\right]dx$$ 3points

$g'(x)=F(x)=0$ Max/Min

$g'(x)=F(x)=$ Slope of $G=0$

$g''(x)=F'(x)=$ Slope of $F(x)$

inflection

$g''(x)=F'(x)=$ Slope of $F(x)=0$ or ϕ

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

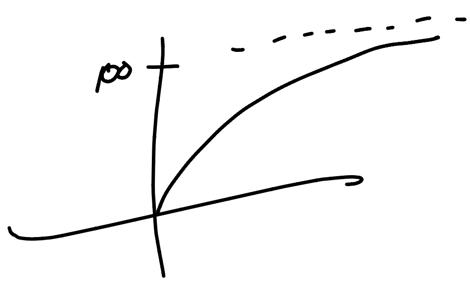
$$\frac{dB}{dt} = \left(20 - \frac{1}{5}B\right)$$

$$\frac{d^2B}{dt^2} = 0 - \frac{1}{5}\frac{dB}{dt} = -\frac{1}{5} \cdot \frac{1}{5}(100 - B) = \frac{-1}{25}(100 - B)$$

$B < 100$

$\frac{-1}{25}$ (Positive #)

$\frac{d^2B}{dt^2}$ = negative ⇒ Concave Down



$$h(x) = g(x) - \frac{1}{2}x^2.$$

$h'(x) = g'(x) - x$

$(x, g'(x))$

0 or ϕ

$g'(x) = x$

$x^2+y^2=2^2$

$x^2+x^2=4$

$2x^2=4$

$x^2=2$

$x=\pm\sqrt{2}$